A Matheuristic for the combined Master Surgical Scheduling and Surgical Cases Assignment problem with Bed Levelling

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Abstract. In this work we address a hierarchical multi-objective combined Master Surgical Scheduling and Surgical Cases Assignment problem with bed levelling and patient priority maximization. To solve the problem we propose a Multi-Neighborhood Local Search based Matheuristic in which several large neighborhoods are sequentially addressed by means of an Integer Programming (IP) model capable to exhaustively explore large neighborhoods in small computational times.

Keywords: MIP based heuristics, Multi Neighborhood Local Search, Operating room planning, Elective surgery, Multi-objective optimisation.

1 Introduction

Operating Rooms (ORs) scheduling and planning can be defined by three hierarchical decisions levels: strategic, tactical and operational that consider respectively the long, medium and short term objectives [18]. The strategic level considers resource allocation problem, determining the number of surgeries, which staff to use for surgeries and defining the amount of the resources available. At tactical level the master surgical schedule, that is the assignment of OR blocks to surgical specialties, is defined together with the number of surgeons, the definition of ward and Intensive Care Unit (ICU) use, and the need of equipment. Finally, at the operational decision level are defined two problems, that is (i) selecting elective patients usually from a long waiting list and assigning them to a specific OR time session (i.e., an operating room open on a specific day) over a planning horizon [8], and (ii) determining the precise sequence of surgical procedures and the allocation of resources for each OR time session [16]. Such problems are further challenged by the inherent stochasticity of their main parameters, such as the surgery duration, the length of stay and the arrival of non-elective patients [9, 10]. In the perspective of considering further resources needed by the patients, bed availability is a topic that recently received a particular attention. Ward bed availability inside a hospital with different surgical specialties is considered in [3, 11] while other studies [16] consider only the use of ICU or Post-Anaesthesia Care Unit (PACU) [19], or both [7].
In order to lead and to evaluate the OR planning decisions, several performance criteria have been reported [4]. Usually, patient priority maximisation and OR

utilisation maximisation are the most used, but also minimise delays and cancellations, maximise patient satisfaction and minimise fixed patient costs or societal costs were considered as objective function for OR planning. On the contrary, the workload balance criteria leads to a smooth – without peaks – stay bed occupancies determining a smooth workload in the ward and, by consequence, an improved quality of care provided to the patients.

Taking into account a patient–centred perspective, a preliminary comparison between two criteria – patient priority maximisation and workload balance – has been reported in [1]. The two criteria provided different results. The patient priority maximisation is a fairness criterion among patients that allowed us to have an OR utilisation close to 100% in all cases. Conversely, the workload balance is a criterion to have a smooth workload along the week, which has been able to schedule a high number of patients in most cases while ensuring a high level of OR utilisation.

In this paper we address the hierarchical multi-objective optimisation combined Master Surgical Scheduling and Surgical Cases Assignment problem with bed levelling and patient priority maximization introduced in [1]. The aim of this work is to develop a fast and efficient tool to solve the above cited problem. We propose a Multi-Neighborhood Local Search based Matheuristic in which several large neighborhoods are sequentially addressed by means of an Integer Programming (IP) model capable to exhaustively explore large neighborhoods in small computational times. The problem description and its mathematical formulation are provided in Section 2. The matheuristic approach is explained in Section 3, while computational results are discussed in Section 4. Finally, conclusions and future developments are reported in Section 5.

2 Problem Statement and Mathematical Model

The problem addressed in this work can be formalised as follows. The goal is to assign simultaneously the OR blocks to a surgical specialty and to schedule patient surgeries in order to maximise a hierarchical objective function considering bed levelling and patients priority.

The general goal of this problem is to maximize the quality of the service. The quality depends on two main factors: waiting times minimization and level of nursing and medical care during the hospitalization. At first sight, these two objectives could seem in contrast, because, the greater the level of occupancy in the department, the lower the level of care the personnel can offer to a single patient. On the other hand, it is necessary to ensure a high level of occupancy to minimize waiting times. But, given an average occupancy rate, a bed levelling over the days of the week guarantee a higher quality of care. Actually, it is more rational to have a smooth distribution of patients in department instead of having a day in which the department is completely full and the day after in which it is almost empty. The solution to achieve the most smoothed global occupancy, i.e. the best bed levelling, is to maximize the bed occupancy in the day and in the department for which it is minimum.

Then, the problem addressed in this work can be formalised as follows. The goal is to assign simultaneously the OR blocks to a surgical specialty and to schedule patient surgeries in order to maximise a hierarchical objective function considering bed levelling and patients priority.

More in details, the primary objective consists in maximising the number of beds occupied in a surgical specialty department, in the day in which the occupation is minimum, which represents the bottleneck of the problem. The secondary objective consists in maximising the global patients satisfaction. To each patient is assigned a score, which is computed as its priority level divided by the waiting time between the diagnosis and the surgery. The global patient satisfaction is defined as the sum of the scores related to the patients which are selected for surgery within the planning horizon.

For each patient are known the surgical specialty to which he/she is assigned, the priority level, the expected length of stay (LOS), the number of days elapsed from the diagnosis, the expected surgery duration. For each specialty is known the number of beds available on each day. Furthermore, the length of each OR block is supposed to be known. The objective is twofold. First, we try to maximise the minimum occupation of beds in a day in a department, secondly to maximise patients satisfaction, as described in the previous paragraph. A patient is assigned to an OR block only if that block has been assigned to the surgical specialty which the patient belongs. The total expected duration of surgeries scheduled in a OR block can not exceed its length. Each scheduled patient occupies a bed in the day of his/her surgery and for a number of following days equal to his/her LOS.

Before reporting the mathematical model, we introduce the following notation. Let I, J and K be respectively the sets of patients, surgical specialties and operating rooms, each indexed by i, j and k. Let $T = \{1,\ldots,N_t\}$ be the set of days in the planning horizon, indexed by t.Let I_j be the subset of patients that belong to specialty j, $j \in J$. For each patient $i \in I$, we are given the expected duration of the surgery p_i, the priority coefficient π_i, and the expected Length of Stay μ_i, expressed in days. Let Φ_{it} be the number of elapsed day between diagnosis of patient i and day t.Note that each OR block in the planning horizon is uniquely defined by the pair of indices (k,t). We denote by s_{kt} the time capacity of the OR session (k,t). Let Λ_{jt} be the number of beds available for specialty j on day t. Finally, let P and M set to $\sum_i \pi_i$ and $\frac{1}{P+1}$, respectively.

Let us introduce the following decision variables: a binary variable X_{ikt} equals to 1 if patient i is assigned to block k on day t, and 0 otherwise; a binary variable Z_{jkt} equals to 1 if block k on day t has been assigned to specialty j, and 0 otherwise; a binary variable Y_{it} equals to 1 if patient i occupies a bed on day t, and 0 otherwise; a binary variable W_it equals to 1 if patient i surgery is scheduled on day t. Let be also O_1 and O_2 the primary and the secondary objective.

$$\max z = O_1 + MO_2 \tag{1a}$$

$$\text{s.t.} \quad \sum_{k\in K}\sum_{t\in T} X_{ikt} \leq 1\,, \qquad i \in I \tag{1b}$$

$$\sum_{i\in I_j} X_{ikt} \leq |I_j| Z_{jkt}\,, \qquad j \in J, k \in K, t \in T \tag{1c}$$

$$\sum_{j\in J} Z_{jkt} \leq 1\,, \qquad k \in K, t \in T \tag{1d}$$

$$\sum_{i\in I} p_i X_{ikt} \leq s_{kt}\,, \qquad k \in K, t \in T \tag{1e}$$

$$W_{it} = \sum_{k\in K} X_{ikt}\,, \qquad i \in I, t \in T \tag{1f}$$

$$\sum_{\tau=t}^{\min(t+\mu_i;N_t)} Y_{i\tau} \geq min(\mu_i+1;N_t-t+1)W_{it}\,, \quad t \in T \tag{1g}$$

$$\sum_{\tau=\max(t-\mu_i,1)}^{t} W_{i\tau} \geq Y_{i\tau}\,, \quad t \in T \tag{1h}$$

$$\sum_{i\in I_j} Y_{it} \leq \Lambda_{jt}\,, \quad t \in T, j \in J \tag{1i}$$

$$O_1 \leq \sum_{i\in I_j} Y_{it}\,, \quad t \in T, j \in J \tag{1j}$$

$$O_2 = \sum_{i\in I}\sum_{t\in T}\sum_{k\in K} \frac{\pi_i}{\Phi_{it}} X_{ikt}\,. \tag{1k}$$

The hierarchical objective function is reported in (1a). The role of the multiplier M is to ensure that if a solution S_1 has a higher value of O_1 with respect to S_2 it would be preferred whichever the correspondent values of O_2. In other words, the secondary objective intervenes in the solutions comparison only when the value of O_1 is exactly the same. Constraint (1b) states that only a subset of patients can be selected from the long waiting list. A patient can be assigned to an OR block only if it is assigned to the surgery specialty to which he/she belongs, as stated in constraint (1c). Constraint (1d) implies that each block must be assigned to at most one specialty. Constraint (1e) imposes that the sum of the surgery times of the patients scheduled in each OR time block (k,t) may not exceed the time block capacity s_{kt}. Constraint (1f) allows to detect whether patient i surgery is scheduled on day t. Constraints (1g) and (1h) imply that, if a patient i is scheduled on day t, he/she will occupy a bed for the following μ_i days. Constraints (1i) limits for each specialty the number of beds occupied each day to the maximum number of available beds. The primary objective function (1j) concerns the maximisation of the number of beds used in the day and the specialty department with the minimal bed usage, which works as bottleneck approach. The max min bed occupation objective function tends also to implicitly fill as much as possible the OR blocks thus avoiding under utilisation of operating rooms. The secondary objective (1k) concerns the maximisation of the patient served multiplied by the relative corresponding patient priority and divided by the waiting days from the diagnosis.

3 A Multi-Neighborhood Local Search Matheuristic

Under the term *Matheuristics* we group all methods in which heuristic or metaheuristic techniques are hybridized with exact methods [15]. Between this broad family we can identify a specific subset of methods in which a MIP or IP model is exploited to analyze large neighborhoods.

The introduction of Large Neighborhood Search, implicitly defined by solution destroy operators [17], requires long computational times to be exhaustively explored, so, usually, simple repairs operator are used to reconstruct a feasible solution starting from a partially destroyed one. This is equivalent to explore only a sample of the solutions contained in a neighborhood with a consequent possible loss of quality. Differently from repair heuristics, the usage of MIP models allow to explore the whole neighborhood exhaustively, i.e. to find the best way to

reconstruct the destroyed solution. This will yield to a much faster convergence toward high quality solutions.
Given a Mixed Integer Programming (MIP) or IP formulation of a problem, a neighborhood generated by a destroy operator can be described adding constraints that fix the value of variables, not involved in the destruction, equal to the value they assumed in the starting solution, while letting the other variables free to assume every value in their domain. The resulting over- constrained version of the model is used to identify the optimal value for the remaining variables. This approach has been proved to be very effective on rich vehicle routing problems, [13] and [14], nurse rostering [6] and jobs scheduling [5].
To solve the problem defined in Section 2, we propose a multi-neighborhood local search matheuristic, which consist in sequentially exploring the following four different neighborhoods.

- **N1: patients assignment reoptimization**. We keep fixed all the ORs assignment to specialties (i.e., the Z variables) and reoptimize only the patient selection and assignment to surgery sessions.
- **N2: 2-days reoptimization**. Given 2 days randomly selected, we keep the other days scheduling fixed and reoptimize the selected days.
- **N3: 2-specialties reoptimization**. Given 2 specialties randomly selected, we keep the other days scheduling fixed and reoptimize the selected days.
- **N4: bed levelling improvement**. This neighborhood is ad-hoc suited to improve the bed levelling and should be applied when the other neighborhoods are only able to improve the second objective but fail to improve the bed levelling, remaining trapped into a local minimum for the primary objective. We identify the *bottleneck* for the bed levelling, i.e. the specialty j and day t for which bed occupation is minimum. We artificially increase the bed availability Λ_{jt} and use as secondary objective function the number of selected patients, trying, in this way, to push the first objective O_1 to be increased. If the obtained solution is not feasible, we reduce Λ_{jt} to its actual value and apply again $N1$.

This procedure is repeated for a fixed number of iterations N_{MAX}. Within the same iteration, each neighborhood n is evaluated α_n times. In order to reduce computational times, we set, for each neighborhood n, an execution time limit for the model, τ_n after which the best obtained solution is reported. Every time we find an improving solution we keep it as current solution.

4 Computational Results

We report a comparison of the computational results obtained by the matheuristic, and those directly obtained by solving the model (1a)-(1k) proposed in Section 2. Two set of instances have been considered. The first, B_2, composed of 8 instances, with 5 ORs, 200 patients uniformly distributed among 4 different specialties and an availability of 20 beds for each specialty. The second one, B_3, composed of 4 instances, with 10 ORs, 400 patients uniformly distributed among 8 different specialties and an availability of 20 beds. More details about the instances generation can be found in [2] in which they are introduced for the first time using the generator reported in [12].
Both the mathematical model and the matheuristic computational have been run under Xpress 7.9. with a CPU time limit of 3600 seconds. Computational test

have been performed on a PC with a 4-core Intel i7-5500U with 2.4GHz CPU and 16 Gb of main memory.

Table 1. Comparison of Matheuristic and Model performances on benchmark set B_2.

INSTANCE	MODEL OF	MODEL secs.	MATHEURISTIC OF	MATHEURISTIC secs.
I200J4B20-1a	13.010	480.28	13.010	108.06
I200J4B20-1b	13.008	444.46	13.008	94.71
I200J4B20-1c	13.008	955.57	13.008	124.93
I200J4B20-1d	13.010	519.12	13.010	131.11
I200J4B20-2a	12.010	53.16	12.010	99.65
I200J4B20-2b	12.011	186.60	12.011	106.90
I200J4B20-2c	12.014	74.10	12.014	119.82
I200J4B20-2d	12.015	24.97	12.015	123.99
average values	12.511	342.28	12.511	113.65

In Table 1, we report, for each instance belonging to set B_2, the objective function (OF) and the computational time in seconds required by the model (MODEL) and by the matheuristic (MATHEURISTIC). As can be noted from the table, the matheuristic always reach the optimal solution with an average percentage saving of the 66.8% on the computational time required.
The same comparison, related to instances belonging to set B_3, is reported in Table 2. On this set, the model is not able to achieve values close to the optimality in any instance within the time limit of 3600 seconds. Therefore, the reported solutions may be suboptimal. The matheuristic sensibly improve the results of the model on all instances within reasonable computational times (524 seconds on average).

Table 2. Comparison of Matheuristic and Model performances on benchmark set B_3.

INSTANCE	MODEL OF	MODEL secs.	MATHEURISTIC OF	MATHEURISTIC secs.
I400J4B20-a	11.010	3600	12.004	567.13
I400J4B20-b	10.010	3600	11.010	522.46
I400J4B20-c	10.011	3600	12.012	529.28
I400J4B20-d	10.013	3600	12.013	477.17
average values	10.261	3600	11.760	524.01

In order to analyze the impact on the algorithm performance of the usage of neighborhood $N4$, specifically developed to address the bed levelling objective, we have compared results obtained by the matheuristics (MH) with those obtained considering only neighborhoods $N1$, $N2$ and $N3$ (MH without $N4$).
Results reported in Table 3 and Table 4 show the crucial role played by the neighborhood $N4$ within the search process. In fact, the absence of $N4$ in the multi-neighborhood local search process negatively affect the method performance. This negative impact, sensibly relevant on $B2$ set, become even stronger on the large instances set $B3$.

Table 3. Analysis of the impact of $N4$ on the matheuristic performance on benchmark set $B2$.

INSTANCE	MH OF	MH secs.	MH without N4 OF	MH without N4 secs.
I200J4B20-1a	13.010	108.06	11.010	42.12
I200J4B20-1b	13.008	94.71	12.009	40.20
I200J4B20-1c	13.008	124.93	11.009	44.54
I200J4B20-1d	13.010	131.11	11.011	41.08
I200J4B20-2a	12.010	99.65	10.011	35.04
I200J4B20-2b	12.011	106.9	10.011	37.85
I200J4B20-2c	12.014	119.82	11.013	37.94
I200J4B20-2d	12.015	123.99	11.016	38.42
average values	12.511	113.65	10.886	39.64

Table 4. Analysis of the impact of $N4$ on the matheuristic performance on benchmark set $B3$.

INSTANCE	MH OF	MH secs.	MH without N4 OF	MH without N4 secs.
I400J4B20-a	12.004	567.13	7.310	182.48
I400J4B20-b	11.010	522.46	6.010	189.70
I400J4B20-c	12.012	529.28	6.311	180.92
I400J4B20-d	12.013	477.17	7.013	178.21
average values	11.760	524.01	6.661	182.83

5 Conclusions

In this paper we propose a hierarchical multi-objective optimisation model for bed levelling and patient priority maximisation for the combined Master Surgical Scheduling and Surgical Cases Assignment problems. The aim of this work is to develop a matheuristic for OR planning and scheduling capable to take into account such different performance criteria.
We propose a multi-neighborhood Local Search based matheuristic in which several large neighborhoods are sequentially addressed by means of an IP model capable to exhaustively explore such large neighborhoods in small computational times. We also developed an ad-hoc neighborhood suited to improve the bed levelling component of the objective function. The computational analysis proved the effectiveness of the proposed matheuristic and, in particularly, the extremely positive impact of the ad-hoc neighborhood.

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